

Two-brane system in a vacuum bulk with a single equation of state

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Abstract. We study the cosmology of a two-brane model in a five-dimensional spacetime, where the extra spatial coordinate is compactified on an orbifold. Additionally, we consider the existence on each brane of matter fields that evolve in time. Solving the Einstein equations in a vacuum bulk, we can show how the matter fields in both branes are connected and they do not evolve independently.

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INTRODUCTION

The idea of brane worlds has been motivated in string theory, in which our visible universe can be seen as a four dimensional "sheet" immersed in a spacetime of more spatial dimensions. Since only three of these spatial dimensions are presently observable, one has to explain why the others are hidden from detection [1, 2, 3]. One such explanation is the so-called Kaluza-Klein (KK) compactification, according to which the size of the extra dimensions is very small (see [4, 5] for a review). In the Horava-Witten (HW) solution [6], gauge fields of the Standard Model of Particle Physics are confined in two 10-branes located at the end points of an S_1/Z_2 orbifold. The 6 extra dimensions on the branes are compactified in a very small scale close to the fundamental one, and their effect on the dynamics is felt through "moduli" fields, i.e., through 5D scalar fields. A 5D realization of the HW theory and the corresponding brane-world cosmology is given in [7, 8, 9]. These solutions can be thought of as effectively 5-dimensional, with an extra dimension that can be large relative to the fundamental scale. They provide the basis for the Arkani-Hamed-Dimopoulos-Dvali (ADD) [10] and Randall-Sundrum (RS) [11, 12] brane models of 5-dimensional gravity. In the RS type 1 model, the space-time necessarily contains two 3-branes, located, respectively, at the fixed points $y = 0$, and $y = y_c$, where y is the fifth spatial dimension. The brane at $y = 0$ is usually called the hidden (or Planck) brane, and the one at $y = y_c$ is called the visible (or TeV) brane.

In the context of two-brane models with matter, it is natural to ask if the evolution in time of the branes is related to one each other (just as in the RS type 1 model, but in general for any metric). Langlois [13] has showed that there exists a relationship between the energy density for the two branes in the form of *cosmological constraints*. The main goal of this paper is to use the relations found by Langlois and analyze the cosmology behind them, generalizing our previous results [14].

MATHEMATICAL BACKGROUND

We begin with the most general 5-dimensional metric in which the branes, located in $y = 0$ ($y = 0$ -brane) and $y = y_c$ ($y = y_c$ -brane), respectively, lie within an homogeneous and isotropic subspace with curvature k for each one, and then

$$ds^2 = -n^2(t, |y|)dt^2 + a^2(t, |y|)g_{ij}dx^i dx^j + b^2(t, |y|)dy^2. \quad (1)$$

We imposed some symmetries upon the model: reflection, $(x^\mu, y) \rightarrow (x^\mu, -y)$, and compactification, $(x^\mu, y) \rightarrow (x^\mu, y + 2my_c)$, $m = 1, 2, \dots$; and we demand on each one of metric coefficients $a(t, |y|)$, $n(t, |y|)$ and $b(t, |y|)$ to be subjected to the conditions [15]:

$$[F']_0 = 2F'|_{y=0+}, \quad (2)$$

$$[F']_c = -2F'|_{y=y_c-}, \quad (3)$$

$$F'' = \frac{d^2 F(t, |y|)}{d|y|^2} + [F']_0 \delta(y) + [F']_c \delta(y - y_c). \quad (4)$$

In the above equations, the prime denotes derivate with respect to y , the square brackets denotes the discontinuity in the first derivative at the positions $y = 0$ and $y = y_c$. Eq. (4) is obtained if we demand that $\frac{d|y|}{dy} = 1$, and that $\frac{d^2|y|}{dy^2} = 2\delta(y) - 2\delta(y - y_c)$, for $y \in [0, y_c]$. A subindex 0 will be used for quantities valued at $y = 0$, whereas a subindex c will be used for quantities valued at $y = y_c$. Now, in order to obtain exact dynamical solutions, we write the five-dimensional Einstein equations, $\tilde{G}_{AB} + \Lambda_5 g_{AB} = \kappa_{(5)}^2 \tilde{T}_{AB}$, for the metric (1),

$$\tilde{G}_{00} = 3\frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right) - 3\frac{n^2}{b^2}\left[\frac{a''}{a} + \frac{a'}{a}\left(\frac{a'}{a} - \frac{b'}{b}\right)\right] + 3k\frac{n^2}{a^2}, \quad (5)$$

$$\begin{aligned} \tilde{G}_{ij} = & \frac{a^2}{b^2}\delta_{ij}\left\{\frac{a'}{a}\left(\frac{a'}{a} + 2\frac{n'}{n}\right) - \frac{b'}{b}\left(\frac{n'}{n} + 2\frac{a'}{a}\right) + 2\frac{a''}{a} + \frac{n''}{n}\right\} \\ & + \frac{a^2}{n^2}\delta_{ij}\left\{\frac{\dot{a}}{a}\left(-\frac{\dot{a}}{a} + 2\frac{\dot{n}}{n}\right) - 2\frac{\ddot{a}}{a} + \frac{\dot{b}}{b}\left(-2\frac{\dot{a}}{a} + \frac{\dot{n}}{n}\right) - \frac{\ddot{b}}{b}\right\} - k\delta_{ij}, \end{aligned} \quad (6)$$

$$\tilde{G}_{05} = 3\left(\frac{\dot{a}n'}{an} + \frac{\dot{b}a'}{ba} - \frac{\dot{a}'}{a}\right), \quad (7)$$

$$\tilde{G}_{55} = 3\frac{a'}{a}\left(\frac{a'}{a} + \frac{n'}{n}\right) - 3\frac{b^2}{n^2}\left[\frac{\ddot{a}}{a} + \frac{\dot{a}}{a}\left(\frac{\dot{a}}{a} - \frac{\dot{n}}{n}\right)\right] - 3k\frac{b^2}{a^2}. \quad (8)$$

We assume that the energy-momentum tensor takes the form

$$\tilde{T}_B^A = \frac{\delta(y)}{b_0} \text{diag}(-\rho_0, p_0, p_0, p_0, 0) + \frac{\delta(y - y_c)}{b_c} \text{diag}(-\rho_c, p_c, p_c, p_c, 0). \quad (9)$$

Using the Bianchi identity, $\nabla_A \tilde{G}_B^A = 0$, we obtain the conservation equation for the energy density in the $y = 0$ -brane,

$$\dot{\rho}_0 + 3\frac{\dot{a}_0}{a_0}(p_0 + \rho_0) = 0. \quad (10)$$

According to the Israel's junction conditions [16], we need to describe the presence of an energy density in terms of a discontinuity in the metric across the origin in the extra spatial coordinate. So, following Wang [15], we obtain the metric coefficients that satisfy the following boundary conditions:

$$\frac{[a']_0}{a_0 b_0} = -\frac{\kappa_{(5)}^2}{3} \rho_0, \quad \frac{[n']_0}{n_0 b_0} = \frac{\kappa_{(5)}^2}{3} (3p_0 + 2\rho_0) = \frac{\kappa_{(5)}^2}{3} \rho_0 (2 + 3\omega_0). \quad (11)$$

$$\frac{[a']_c}{a_c b_c} = -\frac{\kappa_{(5)}^2}{3} \rho_c, \quad \frac{[n']_c}{n_c b_c} = \frac{\kappa_{(5)}^2}{3} (3p_c + 2\rho_c) = \frac{\kappa_{(5)}^2}{3} \rho_c (2 + 3\omega_c). \quad (12)$$

Here, we have assumed a perfect fluid in both branes with $p_0 = \omega_0 \rho_0$ and $p_c = \omega_c \rho_c$. We now proceed to solve the Einstein equations. Integrating Eq. (5), we obtain [17],

$$\left(\frac{a'}{ab}\right)^2 - \left(\frac{\dot{a}}{an}\right)^2 = ka^{-2} - \frac{\Lambda_5}{6} + C_{DR} a^{-4}, \quad (13)$$

where C_{DR} is the called *dark radiation*. On the other hand, Eq. (7), can be written as

$$\frac{\dot{b}}{b} = \frac{n}{a'} \left[\frac{\dot{a}}{n} \right]'. \quad (14)$$

Because of the orbifold symmetry, we are interested in the exact solution of Eqs. (13), and (14) only in the $[0, y_c]$ interval, which we solve in the next section.

EXACT SOLUTIONS FOR A VACUUM BULK

Inspired in a previous work [14], we will find an expression relating the evolution in time for the Hubble parameter in our brane universe, which corresponds to the brane located at $y = y_c$, when the hidden brane at $y = 0$ is dominated by a single matter component.

If we take the ansatz

$$\frac{\dot{a}}{n} = \lambda(t) a^{m/2}, \quad (15)$$

and substitute it in Eqs. (13), and (14), we find that

$$b = a^{m/2}, \quad (16)$$

$$a' = \varepsilon a^{m/2} \left[\lambda^2 a^m + k - \frac{\Lambda_5}{6} a^2 + C_{DR} a^{-2} \right]^{1/2}. \quad (17)$$

When $m = 0, 2, -2$, the λ^2 term behaves as curvature, cosmological constant, and dark radiation, respectively. The term, $\varepsilon = \pm 1$ is the sign of the square root in Eq. (17). For simplicity, we consider in this paper a vacuum bulk and a flat geometry in each brane, $k = \Lambda_5 = C_{DR} = 0$. So, integrating (17) and using the boundary conditions (11) and (12) with aid of Eq. (2), we obtain

$$a(t, y) = a_0(t) \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y \right]^{1/(1-m)}, \quad (18)$$

$$n(t, y) = n_0(t) \left[1 + \left(\frac{m}{2} + 2 + 3\omega_0 \right) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y \right] \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y \right]^{m/(2-2m)}, \quad (19)$$

$$b(t, y) = b_0(t) \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y \right]^{m/(2-2m)}. \quad (20)$$

Notice that for $m = 0$ we recover the linear solutions founded by Langlois [13], and for $m = 2$ and $\omega_0 = -1$, we find a conformal RS metric ¹. We omite here the case $m = 1$.

So far, we have found a family of exact solutions which satisfy the 5D Einstein equations in a vacuum bulk. Imposing on these solutions the boundary conditions (11) and (12), we obtain

$$\rho_c = -\rho_0 \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]^{(m-2)/(2-2m)}, \quad (21)$$

$$\left(\frac{m}{2} + 2 + 3\omega_c \right) = \frac{\left(\frac{m}{2} + 2 + 3\omega_0 \right) \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]}{\left[1 + \left(\frac{m}{2} + 2 + 3\omega_0 \right) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]}. \quad (22)$$

We use in Eq. (22) that $n_c = 1$ and $n_0 = n_0(t)$, and demand that the metric is of the FRW form in the $y = y_c$ -brane. From Eq. (13), and considering the particular case $k = \Lambda_5 = C_{DR} = 0$, we find that

$$\frac{\dot{a}}{an} = \varepsilon \frac{a'}{ab}. \quad (23)$$

Taking into account the boundary conditions (11) and (12), the solutions (19), together with $n_c = 1$ and also Eq. (21), we obtain that

$$H_0 = \frac{\dot{a}_0}{a_0} = -\varepsilon \frac{\kappa_{(5)}^2}{6} n_0 \rho_0 = -\varepsilon \frac{\kappa_{(5)}^2}{6} \rho_0 \frac{\left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]^{-m/(2-2m)}}{\left[1 + \left(\frac{m}{2} + 2 + 3\omega_0 \right) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]}, \quad (24)$$

$$H_c = \frac{\dot{a}_c}{a_c} = +\varepsilon \frac{\kappa_{(5)}^2}{6} n_c \rho_c = -\varepsilon \frac{\kappa_{(5)}^2}{6} \rho_0 \left[1 + (m-1) \frac{\kappa_{(5)}^2}{6} \rho_0 b_0 y_c \right]^{(m-2)/(2-2m)} \quad (25)$$

Eq. (24), together with Eq. (10), gives a solution for $\rho_0(t)$ when ω_0 is a constant, and from it we obtain the evolution in time for $H_c(t)$. The sign ε is chosen such that we obtain an expanding universe within the $y = y_c$ -brane.

¹ Even though it is similar to the metric for a Randall-Sundrum cosmology, we need non-constant values of a_0 , n_0 and b_0 . Note in this case, from Eq. (21), we also have $\rho_0 = -\rho_c$. To recover the RS solutions, we should keep $\Lambda_5 \neq 0$ and $\lambda = 0$ in Eq. (17).

As an example, let us consider the case $m = 0$ and $\omega_0 = \text{const.}$, i. e. a bulk metric that is linear in y , and the $y = 0$ -brane is dominated by a single component. From Eqs. (10), (24) and (25), we obtain

$$1 - X^{-1} + (2 + 3\omega_0)\ln X = \varepsilon (3 + 3\omega_0)T/R \quad \text{and} \quad H_c R = -\varepsilon \frac{X}{1 - X}, \quad (26)$$

where $T = t - t_*$, $X = \rho_0/\rho_{0*}$, $\rho_{0*} = \rho_0(t_*)$, $R = b_0 y_c$ is the radius of compactification, and $\omega_0 \neq -1$. The constant t_* is an epoch in which $\rho_{0*}^{-1} = \kappa_{(5)}^2 b_0 y_c / 6$, i. e. proportional to R . There exist two cases that guarantee positive solutions for H_c in the $y = y_c$ -brane, namely $0 < X < 1$ when $\varepsilon = -1$ and $X > 1$ when $\varepsilon = +1$, which are shown in Fig. 1.

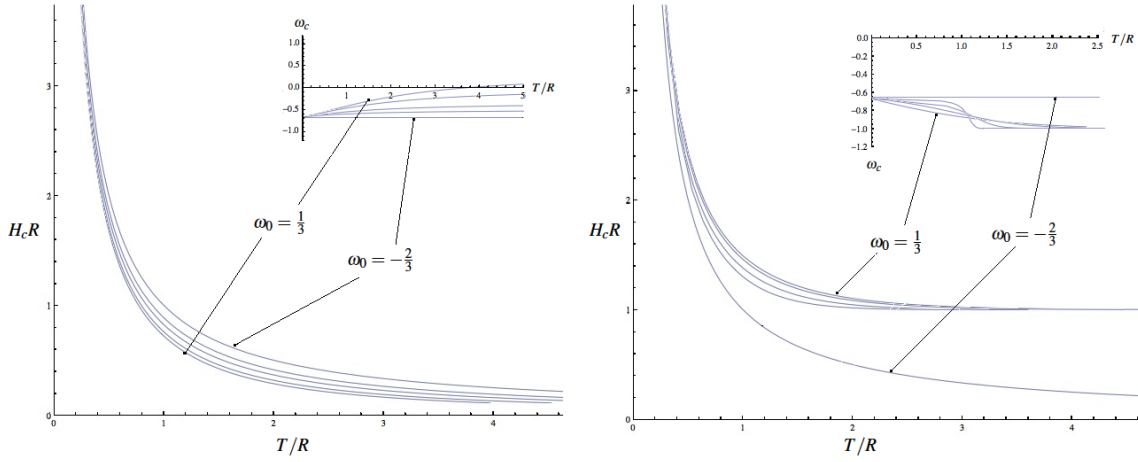


FIGURE 1. Evolution of $H_c R$ as a function of T/R when $m = 0$ and $\omega_0 \in [-2/3, 1/3]$, see Eqs. (26). (Left) The case $0 < X < 1$ and $\varepsilon = -1$; note that $H_c R$ approaches zero when $T/R \gg 1$. (Right) The case $X > 1$ and $\varepsilon = +1$; here $H_c R$ approaches the unity when $T/R \gg 1$, except for $\omega_0 = -2/3$, which is shown separately. The insets in both figures show, from Eq. (22), the evolution of ω_c as a function of T/R in each case.

CONCLUSIONS

We have showed, that in a two-brane system in a 5-D background space-time, there exists a relationship between the cosmologies on the two branes. As in a RS setup, we considered a S_1/Z_2 compactification, and that a FRW metric is recovered in the $y = y_c$ -brane. We were able to generalize the results founded by Langlois et al in[13] and our results are in concordance with a RS type cosmology.

We found an expression relating the energy density in each brane as well as the relation between the equations of state of the branes. Finally, we were able to write the evolution in time for H_c , in the $y = y_c$ -brane, when the $y = 0$ -brane is dominated by a single matter component with a constant equation of state ω_0 . These results may have an interesting interpretation for the cases of dark matter and dark energy in brane universes; we are currently exploring their possibilities, and expect to publish the results elsewhere.

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